

ANSWER KEY TO HOMEWORK SHEET 3

**Functions**

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NOTE: this answer key contains only the correct answers. To get full credit for your solutions, you also need to show the procedure you used to arrive at the correct answer, unless explicitly stated in the exercise.

**Exercise 1.** [15 points]

- (a)  $\text{Ran}(f) = \{ y \mid -1 \leq y \leq 2 \}$ .
- (b)  $\text{Ran}(f) = \{ y \mid -2 \leq y < 0 \text{ OR } 1 < y \leq 3 \text{ OR } 4 \leq y \leq 8 \}$ .
- (c)  $\text{Ran}(f) = \{ y \mid y < -2 \text{ OR } y = 0 \text{ OR } x \geq 3 \}$ .

**Exercise 2.** [12 points]

- (a)  $\text{Ran}(f) = \{ y \mid 0 \leq y \leq \frac{3}{2} \}$ .
- (b)  $\text{Ran}(f) = \{ y \mid -2 \leq y < 0 \text{ OR } 1 < y < \frac{5}{2} \text{ OR } 4 \leq y < 6 \}$ .
- (c)  $\text{Ran}(f) = \{ y \mid -\frac{5}{2} < y < -2 \text{ OR } y = 0 \text{ OR } 3 \leq x < 4 \}$ .

**Exercise 3.** [12 points]

- (a)  $f$  is increasing, hence 1-1.

$$f^{-1}(x) = \begin{cases} \frac{3x+1}{2} & \text{if } -3 \leq x < -1 \\ \frac{2x-3}{5} & \text{if } -1 \leq x \leq 4 \end{cases}$$

- (b)  $f$  is increasing, hence 1-1.

$$f^{-1}(x) = \begin{cases} \frac{3x-8}{5} & \text{if } -4 \leq x < 1 \\ \frac{5x-8}{3} & \text{if } 1 \leq x \leq 4 \end{cases}$$

**Exercise 4.** [21 points]

(a)  $\text{Ran}(f) = \{ y \mid -\frac{7}{2} \leq y \leq 4 \}$ .  $f$  is 1-1.

$$f^{-1}(x) = \begin{cases} 2x + 4 & \text{if } -\frac{7}{2} \leq x < -2 \\ \frac{3}{2}x + 3 & \text{if } -2 \leq x < 0 \\ \frac{1}{2}x + 3 & \text{if } 0 \leq x \leq 4 \end{cases}$$

(b)  $\text{Ran}(f) = \{ y \mid -2 \leq y \leq 2 \}$ .  $f$  is not 1-1.

(c)  $\text{Ran}(f) = \{ y \mid -2 \leq y \leq \frac{7}{2} \}$ .  $f$  is 1-1.

$$f^{-1}(x) = \begin{cases} \frac{1}{2}x - 1 & \text{if } -2 \leq x < 2 \\ x - 2 & \text{if } 2 \leq x < 3 \\ 2x - 5 & \text{if } 3 \leq x \leq \frac{7}{2} \end{cases}$$