Calculus I — Math UN1101
Section 001
New York, 2022/10/26

Answer key to Homework Sheet 8

Derivatives

NOTE: this answer key contains only the correct answers. To get full credit for your solutions, you also need to show the procedure you used to arrive at the correct answer, unless explicitly stated in the exercise.

Exercise 1 (16 points)**.** (a) $f'(x) = 4 - 2x,$, $t_1(x) = 2x + 1$.

(b) $f'(x) = 1 - 3x^2$, $t_1(x) = -2x + 2$.

(c) $f'(x) = 3x^2 - 3$, $t_1(x) = -1$.

(d) $f'(x) = \dfrac{3}{(x+2)^2}$, $t_1(x) = \frac{1}{3}x + \frac{2}{3}$.

(e) $f'(x) = -\dfrac{24}{x^5}$, $t_1(x) = -24x + 30$.

(f) $f'(x) = -\dfrac{\frac{3}{2}\sqrt{x} + x}{x^3}$, $t_1(x) = -\frac{5}{2}x + \frac{9}{2}$.

(g) $f'(x) = -\dfrac{2x}{(1+x^2)^2}$.

(h) $f'(x) = \dfrac{1-x^2}{(1+x^2)^2}$.

Exercise 2 (18 points)**.** (a) x^3.

(b) x^4.

(c) $\dfrac{x^6}{6}$.

(d) $-x$.

(e) $\dfrac{x^3}{3} + \dfrac{3x^2}{2} + x$.

(f) $-\dfrac{1}{x} - \dfrac{1}{2x^2}$.

Exercise 3 (10 points)**.**

(a) 3 real solutions.

$$f(-1) = 5, \qquad f(1) = -3\,,$$
$$\lim_{x\to-\infty} f(x) = -\infty, \qquad \lim_{x\to+\infty} f(x) = +\infty\,.$$

(b) 3 real solutions.

$$f(1) = 1, \qquad f(2) = -1\,,$$
$$\lim_{x\to-\infty} f(x) = -\infty, \qquad \lim_{x\to+\infty} f(x) = +\infty\,.$$

Exercise 4 (16 points.)**.** The function f is

(a) Decreasing between $-\infty$ and $-4-2\sqrt{3}$.

(b) Increasing between $-4-2\sqrt{3}$ and -2.

(c) Increasing between -2 and $-4+2\sqrt{3}$.

(d) Decreasing between $-4+2\sqrt{3}$ and 2.

(e) Decreasing between 2 and $+\infty$.

$$\lim_{x\to-\infty} f(x) = 0, \qquad f(-4-2\sqrt{3}) = -\tfrac{1}{2}+\tfrac{1}{4}\sqrt{3} < 0\,,$$
$$\lim_{x\to-2^-} f(x) = +\infty, \qquad \lim_{x\to-2^+} f(x) = -\infty\,,$$
$$f(-4+2\sqrt{3}) = -\tfrac{1}{2}-\tfrac{1}{4}\sqrt{3}, \qquad \lim_{x\to2^-} f(x) = -\infty\,,$$
$$\lim_{x\to-2^+} f(x) = +\infty, \qquad \lim_{x\to+\infty} f(x) = 0\,.$$

$$\mathrm{Ran}(f) = \{y \;\mid\; y \le -\tfrac{1}{2}-\tfrac{1}{4}\sqrt{3} \quad OR \quad y \ge -\tfrac{1}{2}+\tfrac{1}{4}\sqrt{3}\}$$