COLUMBIA UNIVERSITY
IN THE CITY OF NEW YORK

Calculus I — Math UN1101
Section 001
New York, 2022/11/02

Answer key to Homework Sheet 9

Graphing functions

NOTE: this answer key contains only the correct answers. To get full credit for your solutions, you also need to show the procedure you used to arrive at the correct answer, unless explicitly stated in the exercise.

Exercise 1 (10 points)**.**

(a) $f'(x) = -\dfrac{1}{\sqrt{3-2x}}$.

(b) $f'(x) = \left(1 + \dfrac{1}{\sqrt{x}} + x + 2\sqrt{x}\right) e^x$.

(c) $f'(x) = \dfrac{e^x}{(1-e^x)^2}$.

(d) $f'(x) = \cos x - x \sin x + 2 + 2(\tan x)^2$.

(e) $f'(x) = e^x((\tan x)^2 + \tan x - x)$.

Exercise 2 (4 points)**.**

(a) $f'(x) = 4(5x^6 + 2x^3)^3(30x^5 + 6x^2)$.

(b) $f'(x) = 2x(1 + (\tan(x^2))^2)$.

(c) $f'(x) = 2(\tan x)(1 + (\tan x)^2)$.

(d) $f'(x) = (2x-1)e^{x^2-x}$.

Exercise 3 (16 points)**.**

(a) $\dfrac{1}{6}$.

(b) 12.

(c) $\dfrac{3}{2}$.

(d) 2.

(e) 0.

(f) $-\dfrac{1}{2}$.

(g) $+\infty$.

(h) 0.

Exercise 4 (30 points)**.**

(a) $\mathrm{Ran}(f) = \{\ y \ \mid\ y \leq -1 \quad \text{OR} \quad y \geq 3\ \}$. f is not 1-1.

$$f(0) = -1, \qquad \lim_{x\to 1^-} f(x) = -\infty\,,$$
$$\lim_{x\to 1^+} f(x) = +\infty, \qquad f(4) = 3\,,$$
$$\lim_{x\to +\infty} f(x) = +\infty\,.$$

(b) $\mathrm{Ran}(f) = \{\ y \ \mid\ y \leq -1 \quad \text{OR} \quad y \geq 3\ \}$. f is not 1-1.

$$\lim_{x\to -\infty} f(x) = -\infty, \qquad f(0) = -1$$
$$\lim_{x\to 1^-} f(x) = -\infty, \qquad \lim_{x\to 1^+} f(x) = +\infty\,,$$
$$f(8) = 3, \qquad \lim_{x\to +\infty} f(x) = +\infty\,.$$

(c) $\mathrm{Ran}(f) = \{\ y \ \mid\ y \leq \frac{13}{4}\ \}$. f is not 1-1.

$$\lim_{x\to -\infty} f(x) = -\infty, \qquad f(\tfrac{11}{4}) = \tfrac{13}{4}$$
$$f(3) = 3\,.$$

(d) $\mathrm{Ran}(f) = \{\ y \ \mid\ -2 \le y \le 2\ \}$. f is not 1-1.

$$f(0) = 1, \qquad f(\tfrac{\pi}{2}) = -2$$
$$f(\tfrac{3\pi}{2}) = 2, \qquad f(2\pi) = 1\,.$$

(e) $\mathrm{Ran}(f) = \{\ y \ \mid\ y \ge -\frac{1}{2e}\ \}$. f is not 1-1.

$$\lim_{x\to 0^+} f(x) = 0, \qquad f\left(\frac{1}{\sqrt{e}}\right) = -\frac{1}{2e}$$
$$\lim_{x\to +\infty} f(x) = +\infty\,.$$

(f) $\mathrm{Ran}(f) = \{\ y \ \mid\ y \ge 0\ \}$. f is not 1-1.

$$\lim_{x\to -\infty} f(x) = +\infty, \qquad f(0) = 0$$
$$f(4) = \frac{256}{e^4}, \qquad \lim_{x\to +\infty} f(x) = 0\,.$$