

ANSWER KEY TO HOMEWORK SHEET 10

**Maxima and minima**

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NOTE: this answer key contains only the correct answers. To get full credit for your solutions, you also need to show the procedure you used to arrive at the correct answer, unless explicitly stated in the exercise.

**Exercise 1** (6 points). (a)  $\ln x$ .

(b)  $\frac{1 + \cos x + (\sin x)^2 \cos x}{(\cos x)^2(1 + \cos x)^2}$ .

(c)  $-\frac{x}{\sqrt{1-x^2}} + \arccos(x)$ .

(d)  $\frac{4 \arcsin(2x)}{\sqrt{1-4x^2}}$ .

(e)  $\tan x$ .

(f)  $\frac{\cos(\sqrt{x})}{4\sqrt{x}\sqrt{\sin(\sqrt{x})}}$ .

**Exercise 2** (18 points).

(a)  $\frac{1}{2}x^4 - x^2 + 3x$ .

(b)  $\frac{3}{4}x^4 - x^2 + 3 \ln x$ .

(c)  $\ln x - \frac{1}{2x^2}$ .

(d)  $e^x$ .

(e)  $\frac{2^x}{\ln 2}$ .

(f)  $\tan x - x$ .

(g)  $3 \arctan x$ .

(h)  $x - \arctan x$ .

(i)  $2 \arcsin x$ .

**Exercise 3** (12 points).

- (a) Absolute minimum  $f(0) = 5$ , absolute maximum  $f(3) = 113$ .
- (b) Absolute minimum  $f(0) = -64$ , absolute maximum  $f(3) = 125$ .
- (c) Absolute minimum  $f(0) = 0$ , absolute maximum  $f(1) = 1$ .

**Exercise 4** (24 points).

- (a) Increasing between  $-\infty$  and 1 and between 2 and  $+\infty$ , decreasing between 1 and 2. Local maximum at 1, local minimum at 2. No absolute maximum or minimum.
- (b) Increasing between  $-1$  and 1, decreasing between  $-\infty$  and  $-1$  and between 1 and  $+\infty$ . Absolute and local maximum at 1, absolute and local minimum at  $-1$ .
- (c) Increasing between  $\frac{\pi}{2}$  and  $\frac{3\pi}{2}$ , decreasing between 0 and  $\frac{\pi}{2}$  and between  $\frac{3\pi}{2}$  and  $2\pi$ . Absolute and local minimum at  $\frac{\pi}{2}$ , absolute and local maximum at  $\frac{3\pi}{2}$ .
- (d) Increasing between  $\frac{1}{\sqrt{e}}$  and  $+\infty$ , decreasing between 0 and  $\frac{1}{\sqrt{e}}$ . No absolute or local maximum, absolute and local minimum at  $\frac{1}{\sqrt{e}}$ .
- (e) Increasing between 0 and 4, decreasing between  $-\infty$  and 0 and between 4 and  $+\infty$ . No absolute maximum, local maximum at 4, absolute and local minimum at 0.
- (f) Increasing between 0 and  $+\infty$ , decreasing between  $-\infty$  and 0. No absolute or local maximum, absolute and local minimum at 0.