COLUMBIA UNIVERSITY
IN THE CITY OF NEW YORK

ANSWER KEY TO HOMEWORK SHEET 11
Concavity

NOTE: this answer key contains only the correct answers. To get full credit for your solutions, you also need to show the procedure you used to arrive at the correct answer, unless explicitly stated in the exercise.

Exercise 1 (16 points).

(a) $\cos(\sin(\sin(x)))\cos(\sin(x))\cos(x)$.

(b) $(1 + (\tan(\sin(x)\cos(x))^2)(\cos(x)^2 - \sin(x)^2)$.

(c) $(1 + (\tan(\sin(x)\cos(\cos(x))))^2)(\cos(x)\cos(\cos(x)) + \sin(x)^2\sin(\cos(x)))$.

(d) $\dfrac{\ln(x)+1}{x\ln(x)}$.

(e) $4\left(\dfrac{x-1}{x^2+x+1}\right)^3 \dfrac{2+2x-x^2}{(x^2+x+1)^2} = \dfrac{4(x-1)^3(2+2x-x^2)}{(x^2+x+1)^5}$.

(f) $-\dfrac{(x^2+1)^3(9+56x+x^2)}{(2x+1)^4(3x-1)^6}$.

(g) $\dfrac{\ln(x^4)+4}{2\sqrt{x\ln(x^4)}} = \dfrac{\ln(x)+1}{\sqrt{x\ln(x)}}$.

(h) $\dfrac{1}{2x\ln(\sqrt{x})}$.

Exercise 2 (24 points).

(a) CU between $\frac{3}{2}$ and $+\infty$, CD between $-\infty$ and $\frac{3}{2}$. Inflection point at $\frac{3}{2}$.

(b) CU between $-\sqrt{3}$ and 0 and between $\sqrt{3}$ and $+\infty$, CD between $-\infty$ and $-\sqrt{3}$ and between 0 and $\sqrt{3}$. Inflection points at $-\sqrt{3}, 0, \sqrt{3}$.

(c) CU between $\frac{\pi}{6}$ and $\frac{5\pi}{6}$, CD between 0 and $\frac{\pi}{6}$ and between $\frac{5\pi}{6}$ and 2π. Inflection points at $\frac{\pi}{6}, \frac{5\pi}{6}$.

(d) CU between $\frac{1}{\sqrt{e^3}}$ and $+\infty$, CD between 0 and $\frac{1}{\sqrt{e^3}}$. Inflection point at $\frac{1}{\sqrt{e^3}}$.

(e) CU between $-\infty$ and 2 and between 6 and $+\infty$, CD between 2 and 6. Inflection points at $2, 6$.

(f) CU between -3 and 3, CD between $-\infty$ and -3 and between 3 and $+\infty$. Inflection points at $-3, 3$.

Exercise 3 (20 points).

(a)

$$\lim_{x\to-\infty} f(x) = 1, \qquad\qquad f(0) = -1,$$
$$\lim_{x\to+\infty} f(x) = 1.$$

CU between $-\frac{2}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$, CD between $-\infty$ and $-\frac{2}{\sqrt{3}}$ and between $\frac{2}{\sqrt{3}}$ and $+\infty$. Inflection points at $\pm\frac{2}{\sqrt{3}}$.

(b)

$$\lim_{x\to-\infty} f(x) = 0, \qquad\qquad \lim_{x\to 0^-} f(x) = +\infty,$$
$$\lim_{x\to 0^+} f(x) = -\infty, \qquad\qquad \lim_{x\to+\infty} f(x) = -1.$$

CU between $-\infty$ and 0, CD between 0 and $+\infty$. No inflection points.

(c)

$$\lim_{x\to 0^+} f(x) = +\infty, \qquad\qquad f(1) = \frac{5}{6},$$
$$f(2) = \frac{4}{3} - \frac{2}{3}\ln 2, \qquad\qquad \lim_{x\to+\infty} f(x) = -\infty.$$

CU between 0 and $\sqrt{2}$, CD between $\sqrt{2}$ and $+\infty$. Inflection point at $\sqrt{2}$.

(d)

$$\lim_{x\to-\infty} f(x) = e^{-\frac{\pi}{2}}, \qquad\qquad \lim_{x\to+\infty} f(x) = e^{\frac{\pi}{2}}.$$

CU between $-\infty$ and $\frac{1}{2}$, CD between $\frac{1}{2}$ and $+\infty$. Inflection point at $\frac{1}{2}$.