COLUMBIA UNIVERSITY
IN THE CITY OF NEW YORK

Calculus I — Math UN1101
Section 001
New York, 2022/10/19

HOMEWORK SHEET 7

Limits and continuous functions

Exercise 1 (14 points)**.** Find the following limits.

(a) $\lim_{x\to+\infty} \arctan(-x) + e^{-x}$.

(b) $\lim_{x\to-\infty} e^{x^2}$.

(c) $\lim_{x\to-\infty} e^{8+2x-x^3}$.

(d) $\lim_{x\to+\infty} e^x + \cos x + \sin x$.

(e) $\lim_{x\to+\infty} e^{-x}\cos x$.

(f) $\lim_{x\to-\infty} \arctan(7 - x + 3x^5)$.

(g) $\lim_{x\to+\infty} \ln\left(\frac{3x^4 - 8}{2 + x^2}\right)$.

Exercise 2 (15 points)**.** Find the following limits:

(a) $\lim_{x\to 0^+} x^2 e^{\sin\left(\frac{1}{x}\right)}$.

(b) $\lim_{\theta\to 0^+} \dfrac{\sin\theta}{\theta}$.
(Hint: you can use the following well known inequalities.
$\sin\theta \le \theta$; and for $0 < \theta < \frac{\pi}{2}$, $\theta < \tan\theta$.)

(c) $\lim_{x\to 0^+} \dfrac{2-\cos\left(\frac{1}{x}\right)}{\frac{1}{x}+3}$.

(d) $\lim_{x\to 0^+} \dfrac{\left(\cos\left(\frac{2}{x}\right)\right)^2}{3-\frac{2}{x}}$.

(e) $\lim_{h\to 0} \dfrac{\sin(x+h)-\sin(x)}{h}$.
(Hint: you can use the trigonometric formula
$\sin\theta - \sin\phi = 2\sin\left(\frac{\theta-\phi}{2}\right)\cos\left(\frac{\theta+\phi}{2}\right)$.)

Exercise 3 (7 points)**.** Verify that the equation

$$x^2 - \pi^2 \sin x + \frac{\pi^2}{4} = 0$$

has at least one solution.

Exercise 4 (12 points)**.** Sketch the graph of an example of a function f that satisfies simultaneously the 7 given conditions below.

1. $\mathrm{Dom}(f) = \{x \mid -3 < x < 3\}$.
2. f is continuous.
3. f is increasing between -3 and 0.
4. f is decreasing between 0 and 3.
5. $\lim\limits_{x \to -3^+} f(x) = -\infty$.
6. $f(0) = 5$.
7. $\lim\limits_{x \to 3^-} f(x) = 2$.

Exercise 5 (12 points)**.** Consider the following function:

$$f(x) = \frac{x^2 + x + \frac{1}{4}}{x^2 - 4} \qquad\qquad \mathrm{Dom}(f) = \{x \mid x \neq \pm 2\}\,.$$

Assume that f satisfies the following 5 conditions:

(a) Decreasing between $-\infty$ and -8.

(b) Increasing between -8 and -2.

(c) Increasing between -2 and $-\frac{1}{2}$.

(d) Decreasing between $-\frac{1}{2}$ and 2.

(e) Decreasing between 2 and $+\infty$.

Sketch the graph of f.

Submission

Homework sheets are published online every Wednesday night. You must submit your written solutions 6 days later, on the night between Tuesday and Wednesday. The deadline is on Wednesday early morning, at 5am. Submission is online, via Courseworks at the Gradescope page.

You will need to produce a pdf file with your solutions. In the pdf file, please make sure that every page contains material relevant to only one exercise, and mark the exercise number clearly on top of the page.

To produce the pdf file, you can write on paper and then scan your handwritten solutions. If you don't have a scanner, you can use your phone, but please download and use a scanning app. Don't just take pictures, the quality of the pictures is not good enough. A scanning app makes scans of a much better quality, and converts automatically to pdf. Remember that scanning is time-consuming, please do this well in advance of the dead-line.

If you prefer to directly write electronically, e.g. with a stylus on a tablet, this is totally fine too. You just need to produce a pdf.

When you have produced your pdf file, go to the Gradescope page on Courseworks, there you can upload your pdf file.

Grading

Every homework sheet is worth 60 points. At the end of the term we will compute the average of the grades you got at the homework sheets. This average will contribute to your final grade.

If everything goes as planned, there will be 13 graded homework sheets during the term.

If you don't manage to submit your homework on time, if you are just a few days late, please, submit anyway. We will accept late submissions, but we deduct 10% of the points for every day of lateness.

Rules for working at your homework problems

You are welcome to work together with other students in the class, possibly in little groups. But, at the end, you have to write your final solutions by yourself, in your own words. Sharing complete written solutions with other students in the class will be considered as cheating, as well as copying them.

The homework problems require you to explain your answer. You have to show the procedure and the reasoning you follow to arrive at the solution. In some cases, writing mathematical formulae is not enough, and you will need to add some written sentences to explain what you are doing. In these cases, please try to use complete sentences, not just isolated words. This exercise is very useful for you to understand what you are doing, and it prepares you for the midterms and final exams.

How to approach the homework

You have 6 days time to solve every homework sheet. I recommend you to work a little bit every evening, instead of doing everything the last afternoon. In the first three days, you should work by yourself, trying to solve the problems alone. Take note of all the things you don't understand, and the questions that arise. During the last three days, it is the best moment to discuss the problems with your classmates. You can ask for their help for the few things you didn't manage to do by yourself.

But it is very important that you first try to solve the exercises alone, before asking for help. During the midterms and the final exams you will be alone, and you have to prepare for that.