COLUMBIA UNIVERSITY
IN THE CITY OF NEW YORK

Calculus I — Math UN1101
Section 001
New York, 2022/12/12

Mock Final Exam

This mock exam can be a useful tool to prepare for the final exam. To get the most out of this Mock Exam, take 3 hours time, and solve the problems during this time, by yourself, without using calculators nor asking for help. Write down your solutions completely, as if it were the real exam. This will give you a good idea about how you can perform during the final exam.

Notice that this exam is not identical to the final exam. Don't expect to see exactly the same exercises and the same questions at the final. This exam is only one possible way I could write the final exam, but other possibilities exist. And you will see one of them during the final exam.

Exercise 1 (8 points)**.** Find the following limits:

(a) $\lim_{x\to+\infty} \dfrac{\ln(x^2)+1}{\ln(x)-2}$.

(b) $\lim_{x\to0^-} \dfrac{1}{x} - \dfrac{1}{x^2+x}$.

(c) $\lim_{x\to0^-} \dfrac{x^2}{1-\cos x}$.

(d) $\lim_{x\to0^-} \dfrac{\ln(x^2)}{x^3}$.

Exercise 2 (4 points)**.** Count the number of real solutions of the following equation

$$x^3 - 3x + 3 = 0.$$

Exercise 3 (16 points)**.** Draw the graph of the following function:

$$f(x) = \frac{x^2 - 5x + 4}{x - 5}.$$

Then compute the range of f.

Exercise 4 (16 points)**.** Let f be the following function:

$$f(x) = \tfrac{1}{2}x^4 - 4x^2 + 3, \qquad \mathrm{Dom}(f) = [-3, 3]\,.$$

Find the intervals where f is increasing or decreasing, the absolute and local maxima and minima, the intervals of concavity and the inflection points.

Exercise 5 (16 points)**.** Compute the following definite integrals

(a) $\displaystyle\int_0^1 (x+1)^3 \, dx.$

(b) $\displaystyle\int_0^1 \frac{1}{x^2+1} \, dx.$

(c) $\displaystyle\int_0^1 \frac{x}{x^2+1} \, dx.$

(d) $\displaystyle\int_1^2 x^4 \ln x \, dx.$

(e) $\displaystyle\int_0^1 \frac{1}{(1+\sqrt{x})^4} \, dx.$

Exercise 6 (8 points)**.** Compute the area of the plane region:

$$\{(x, y) \mid x^2 \leq y \leq x + 2\}$$