COLUMBIA UNIVERSITY
IN THE CITY OF NEW YORK

Number Theory and Cryptography
Math UN3020
New York, 2023/01/18

EXERCISE SHEET 1

Numbers

Exercise 1 (27 points)**.**
The operations $+, \cdot$ on the number sets $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$ satisfy the following properties:

(a) $\forall a, b, c,\ (a+b)+c = a+(b+c)$ (associativity of $+$).

(b) $\forall a, b,\ a+b = b+a$ (commutativity of $+$).

(c) $\forall a,\ a+0 = 0+a = a$ (additive identity).

(d) $\forall a,\ \exists k$ s.t. $a+k = 0$ (opposite).

(e) $\forall a, b, c,\ (ab)c = a(bc)$ (associativity of $\cdot$).

(f) $\forall a, b,\ ab = ba$ (commutativity of $\cdot$).

(g) $\forall a,\ a \cdot 1 = 1 \cdot a = a$ (multiplicative identity).

(h) $\forall a, b, c,\ a(b+c) = ab + ac$ (distributivity).

Moreover, the operation $\cdot$ on the number sets $\mathbb{Q}, \mathbb{R}, \mathbb{C}$ also satisfy the following property:

(i) $\forall a \neq 0,\ \exists k$ s.t. $ak = 1$ (inverse).

Choose distinct numbers a, b, c, and verify that all the properties stated above are true for the chosen numbers.

Exercise 2 (13 points)**.** Among the properties stated in the previous exercise, which ones hold and which ones don't hold for the operations $+, \cdot$ on the number set $\mathbb{N}$?

Exercise 3 (20 points)**.** Which of the following statements are true, which are false?

(a) $4 \mid 5$.

(b) $4 \mid 12$.

(c) $3 \nmid 9$.

(d) $3 \nmid 10$.

(e) $12 \mid 2$.

(f) $8 \nmid 4$.

(g) $1 \mid 5$.

(h) $0 \mid 6$.

(i) $2 \mid 1$.

(j) $51 \mid 0$.