EXERCISE SHEET 6

Linear Congruences

Exercise 1 (12 points.)**.** Show that, for all $m > 0$, we have

$$\gcd(ma, mb) = m \gcd(a, b).$$

Exercise 2 (9 points)**.** Find all the zero-divisors on $\mathbb{Z}_{18}$. For each zero-divisor, find a non-zero element of $\mathbb{Z}_{18}$ that multiplies to zero with the zero-divisor.

Exercise 3 (15 points)**.** Solve the following linear congruences.

(a) $131x \equiv 11 \pmod{1979}$.

(b) $127x \equiv 11 \pmod{1091}$.

(c) $3x \equiv 12 \pmod{21}$.

(d) $6x \equiv 12 \pmod{15}$.

(e) $2x \equiv 7 \pmod{10}$.

Exercise 4 (15 points). Solve the following systems of congruences.

(a)
$$\begin{cases} x \equiv 4 & (\text{mod } 55) \\ x \equiv 11 & (\text{mod } 69) \end{cases}$$

(b)
$$\begin{cases} x \equiv 5 & (\text{mod } 11) \\ x \equiv 7 & (\text{mod } 13) \end{cases}$$

(c)
$$\begin{cases} x \equiv 11 & (\text{mod } 16) \\ x \equiv 16 & (\text{mod } 27) \end{cases}$$

Exercise 5 (9 points). Solve the following system of congruences.

$$\begin{cases} 3x \equiv 6 & (\text{mod } 9) \\ 3x \equiv 2 & (\text{mod } 7) \\ x \equiv 1 & (\text{mod } 5) \end{cases}$$